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PRESSURE DROP AND HEAT TRANSFER IN AIRFLOW COOLED FINNED HEAT SINKS USING CORRELATIONS

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Abstract. *A network flow model was developed to calculate the pressure drop across a finned parallel plate heat sink. Airflow was assumed at either laminar or turbulent regime in the fin channels. The pressure drop included viscous friction and the contraction and expansion losses of the flow. Heat transfer correlations were adapted from the literature. A comparison was made with published data for the airflow volumetric flow rate across a heat sink and the associated thermal resistance.*

Keywords: airflow, heat sink, rectangular channels, correlations.

1. INTRODUCTION

Heat generation in electronic devices has increased in the last years due to advances in the microelectronic technology, so it became necessary to optimize the heat sink design as shown in Fig. 1. Empirical and theoretical correlations are valuable tools at the early stages of heat sinks design. Those correlations can be coded in any programming language so that predictions about the fin array behavior are available after a short time period. However, specific correlations for fins systems do not exist. Therfore, the correlations available in the literature may result in significant imprecision for some geometric configurations of the rectangular fins system.

In a pioneer work, Tuckerman & Pease (1981) showed that an array of small fins attached to the surface of an electronic component with a liquid flowing through them can dissipate large amounts of heat. They observed that when interfin flow was in laminar regime, the heat transfer coefficient depended inversely on the channel width, whereas the cooling fluid flow rate depended directly on such width.

Goldberg (1984) tested three different fin arrays using the environment air as cooling fluid. Each array had different fin thickness with the channel width always the same. The fluid flow through each fin array was made constant and equal to 30 l/min. The fin array geometry with the largest pressure drop (smaller channel width) also had the smallest thermal resistance. The author used a methodology similar to that of Tuckerman & Pease (1981) to calculate the best ratio of fin thickness to channel width to achieve the smallest thermal resistance. He found in the same way that the ratio should be equal to one.

Bar-Cohen & Jelinek (1985) studied the optimization of rectangular fin arrays cooled by airflow. They intended to maximize the rate of heat removal or to minimize the amount of the heat sink material. The fin array material, the airflow rate, the available pressure drop and the fins length were specified and the remaining dimensions, the fins height or the fins number, were calculated by a computational procedure presented in their paper.

Knight et al. (1991) published the first paper treating a widespread optimization scheme for rectangular fin arrays under forced airflow convection using generalized dimensionless variables. They showed the existence of a number of fins and fin thickness to channel width ratio that minimizes the thermal resistance of the array. In their analysis, the completely developed flow condition was still assumed, but other restrictions were relaxed as the possibility of turbulent flow regime in the interfin channels. Their results were compared to those from Tuckerman & Pease (1981) and Goldberg (1984). Significant reductions of the thermal resistance were found (about 30%).

In a later work, Knight et al. (1992a) improved the optimization procedure including the effects of the development of the flow velocity profile, choosing better correlations to calculate the pressure drop and heat transfer effects.

Moreover, Knight et al. (1992b) presented experimental evidences that the developed methodology leads to the optimal design of the fin array. They tested three arrays of aluminum fins with identical external dimensions but each one with different number of fins. The airflow through the channels was adjusted to provide the same pressure drop and the thermal resistance of each array was measured. The best predicted geometry had eight fins while the other two had five and eleven fins. Compared to the best, the measured thermal resistances of these assemblies were respectively 19% and 13% larger.

2. ANALYSIS

Figure 2 represents the flow within a duct with sudden contraction at the heat sink inlet (section 1), viscous friction inside the rectangular ducts of the fins and sudden expansion at the heat sink outlet (section 2). The base of fins is considered flush mounted with the duct wall without tip clearance. This characterizes a confined flow condition.

Kays & Perkins (1985) presented a general relationship for the total static pressure drop corresponding to the model presented in Fig. 1

$$\Delta P_T = \frac{\rho_1 U_m^2}{2}\left[\left(1-\sigma^2+K_c\right)+2\left(\frac{\rho_1}{\rho_2}-1\right)+4f\,\frac{L}{D}\frac{\rho_1}{\bar{\rho}}-\left(1-\sigma^2-K_e\right)\frac{\rho_1}{\rho_2}\right] \tag{1}$$

In Eq. (1), the terms on right side represent respectively the entrance effect, the flow acceleration effect for a compressible fluid, the friction pressure drop, and the exit effect. The variables K_c and K_e are pressure drop coefficients at the inlet and outlet sections respectively and σ is the free flow / frontal area ratio ($\sigma = 1$ for a single duct). The mean velocity U_m is based on the narrowest section of the flow and ρ is the density of the fluid.

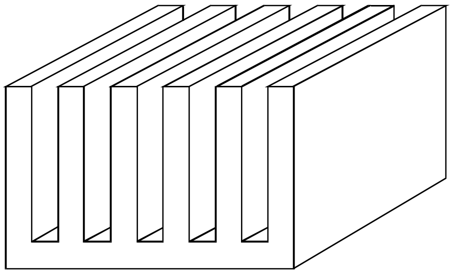

Fig. 1 – Longitudinal and parallel fins heat sink

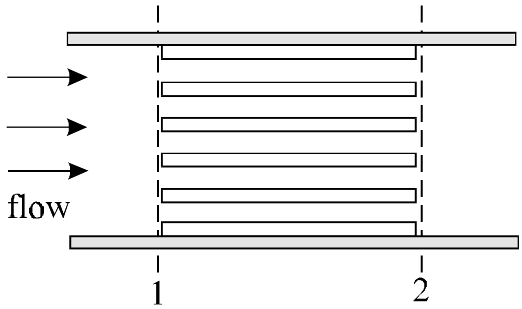

Fig. 2 - Model of the pressure drop

If the flow is incompressible, the density of fluid (ρ) remains constant and Eq. (1) becomes

$$\Delta P_T = \frac{\rho U_m^2}{2}\left[K_c + 4f\frac{L}{D} + K_e \right] \tag{2}$$

2.1 Pressure drops due to sudden contractions and expansions

For incompressible or low Mach number flows, the mechanical energy losses in abrupt section changes are calculated using the expansion K_e and contraction K_c coefficients. Kays (1950) defined them as

$$\Delta P_e = \rho K_e \frac{U_m^2}{2} \tag{3}$$

$$\Delta P_c = \rho K_c \frac{U_m^2}{2} \tag{4}$$

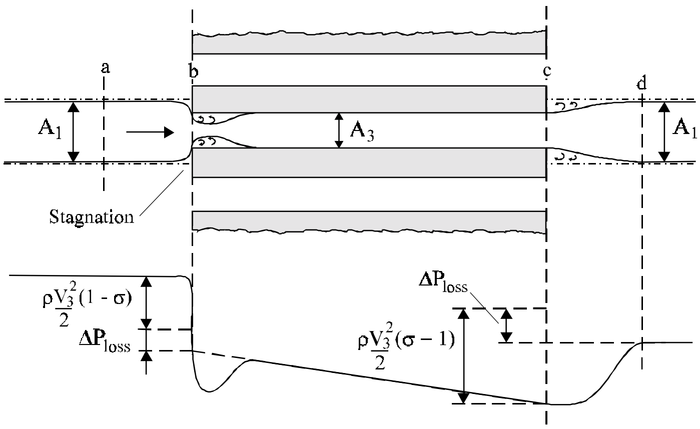

Fig. 3 – System diagram and pressure variation in sudden contraction and expansion

At sudden expansions there is a pressure variation caused only by the flow expansion effects, while at sudden contractions a section flow contraction occurs at the inlet (the venna-contracta) and a posterior flow expansion within the fin channels (see Fig. 3). The entrance effect is regarded including both pressure losses due to the deceleration near to the stagnation point and re-expansion of the flow. The expansion coefficient K_e is a function only of the area ratio σ, while the contraction coefficient K_c is strictly empiric.

Kays (1950), considering laminar and turbulent flow regimes presented values of K_e and K_c for a flat duct and a square section duct in graphs. In this work those data were interpolated through polynomial functions with the following considerations (see Fig. 4 and 5). In laminar flow regime, the coefficients K_e and K_c depend only on σ. However, in turbulent regime they depend both on σ and the Reynolds number based on the hydraulic diameter (Re). The dependence on Re is weak (variation about 10% for Re varying from 2,000 to infinite) and mean values of K_e and K_c were used in turbulent flow condition.

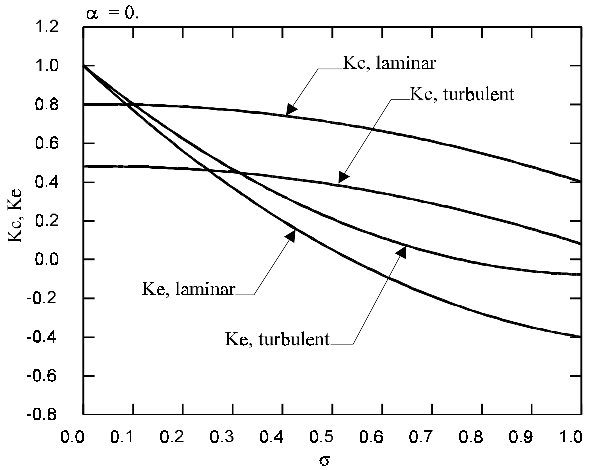

Fig. 4 - Contraction and expansion coefficients for flat duct

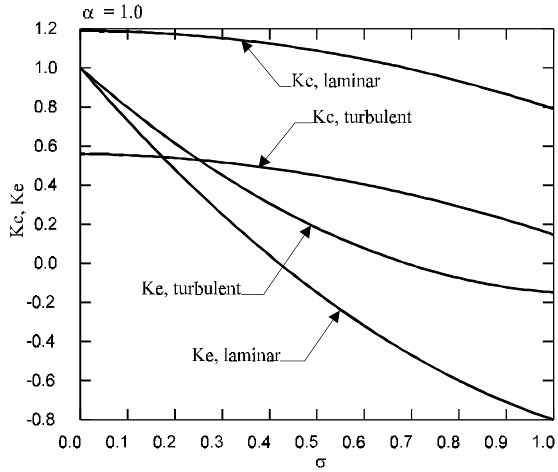

Fig. 5 – Contraction and expansion coefficients for square ducts

The aspect ratio α is defined as the ratio of the smallest to the largest dimension of the rectangular duct cross section. Parallel plates (flat duct) and ducts of square section represent the two extreme cases among ducts of rectangular section ($\alpha = 0$ and $\alpha = 1$ respectively).

The K_e and K_c values for a duct with an aspect ratio $0 < \alpha < 1$ was made having an intermediary value between those for a flat and a square duct. They were obtained using linear interpolation functions according to Eq. (5) and the values presented in Table 1. In that equation, when α approaches zero the values of K_e and K_c approach those for a flat duct. The inverse occurs when α approaches 1 and the coefficients approach those for a square section duct.

The flow is considered in laminar regime if Re < 2,000, otherwise it is turbulent.

$$K_{e,c} = (1-\alpha)\left(A_0 - A_1\,\sigma + A_2\,\sigma^2\right) + \alpha\left(B_0 - B_1\,\sigma + B_2\sigma^2\right) \quad (5)$$

TABLE 1- Coefficients for Equation (5)

	Contraction		Expansion	
	Laminar	Turbulent	Laminar	Turbulent
A	0.800	0.480	1.000	1.000
A	0.029	0.029	-2.400	-2.083
A	-0.430	-0.430	1.000	1.005
B	1.190	0.560	1.000	1.000
B	-0.011	-0.030	-2.800	-2.125
B	-0.389	-0.383	1.000	0.976

It is important to notice that from the laminar flow to the turbulent, K_c decreases while K_e increases. A large variation in the magnitude of these coefficients occurs when the flow regime changes.

2.2 Pressure drops associated to viscous effects

This portion of pressure drop is due to the viscous fluid friction in the walls of the fins rectangular channels. The pressure drop ΔP_f was calculated as follows:

$$\Delta P_f = 4\left(f \frac{x}{D_h} \right)\frac{1}{2}\rho U_m^2 \tag{6}$$

The parameter f represents the Fanning friction factor based on the mean wall shear, x is the length from the duct entrance to the calculation point, U_m is the mean flow velocity and D_h, the duct hydraulic diameter.

Kays & Perkins (1985) presented three kinds of friction factor: f, f_x and f_{app}. The coefficient f_x is termed the local friction factor based on the local wall shear stress at the x position. The coefficient f is based on the mean shear from x = 0 to x (the length from the channels inlet). The development of the velocity profile causes pressure drop due to the increase of the total momentum flux of the fluid flow. Moreover, the pressure drop calculation must take into account any variation in the momentum flux, as well as the effects of the surface shear forces. The combination of these effects is then incorporated in a single mean friction factor called apparent friction factor f_{app}

If the flow regime is laminar with developed velocity profile, for a duct of arbitrary section, f_∞ is given by $f_\infty = C / Re$. The constant C depends on the cross section geometry of the duct. For ducts of circular cross section, C = 16 and for parallel plates, C = 24 (Kays & London (1984)).

Jones (1976) recommends that if the flow has developed velocity profile within rectangular cross section ducts with aspect ratio $\alpha \leq 1$, an equivalent diameter D_{eq} should be used in the expression of f_∞, called the laminar equivalent diameter $D_{eq} = \phi\, D_h$ with,

$$\phi = \frac{2}{3} + \frac{11}{24}\alpha\,(2-\alpha) \tag{7}$$

The value of f_∞ for rectangular ducts with any aspect ratio α can be calculated by using the equation of ϕ considering $Re_{eq} = \phi\, Re$ and $f_\infty = 16 / (\phi\, Re)$.

Shah & London (1978) presented an expression for f_{app} as a function of the dimensionless length $x^+ = (x / D_h)/Re$. They considered a circular cross section duct, uniform velocity profile at the duct inlet and laminar flow regime

$$f_{app}\, Re = \frac{3.435}{(x^+)^{0.5}} + \frac{16 + 1.25\ (4x^+)^{-1} - 3.435\ (x^+)^{-0.5}}{1 + 0.00021\,(x^+)^{-2}} \tag{8}$$

A similar concept to that from Jones (1976) was used in this work to adjust Eq. (8) in order to obtain values for rectangular ducts.

$$f_{app}\,Re = \frac{3.435}{(x^+)^{0.5}} + \frac{16\,\phi^{-1} + 1.25\,(4x^+)^{-1} - 3.435\,(x^+)^{-0.5}}{1 + 0.00021\,(x^+)^{-2}} \tag{9}$$

Numerical results of $f_{app}Re$ for rectangular ducts with several aspect ratios α, laminar flow regime and uniform velocity profile at the duct inlet, obtained by Curr et al. (1972), were presented in a graph by Shah & Bhatti (1987). These results were compared to those values calculated by using Eq. (9) and presented in Fig. 6. An excellent agreement of the $f_{app}Re$ values was observed (percentile difference less than 1%).

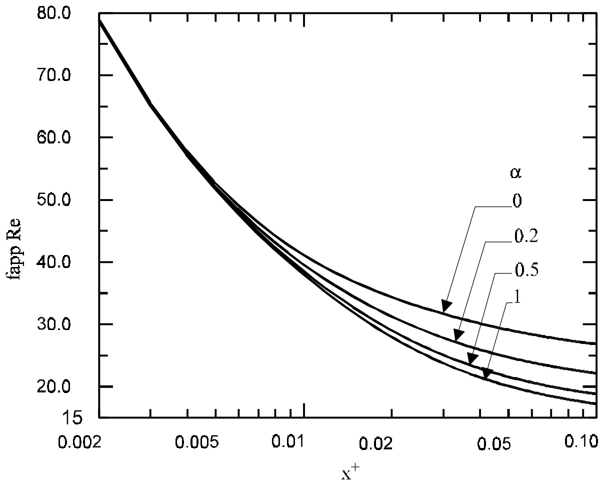

Fig. 6 – Apparent friction factor for laminar flow within rectangular ducts

The inferior limit of the critical Reynolds number (Re_c) has been established through experimental investigations for ducts of rectangular cross section. The configuration of the duct entrance has a strong influence on the Re_c behavior as well as the aspect ratio α. Davis & White (1928) presented the experimental measures of Re_c considering an abrupt duct entrance and various aspect ratios α between 0 and 1. Those data were used in this work to determine a polynomial interpolation function of Re_c in function of α.

$$Re_c = 3{,}035.22 - 4{,}497.45\alpha + 10{,}719.4\alpha^2 - 11{,}285.3\alpha^3 + 4{,}232.46\alpha^4 \tag{10}$$

Phillips (1990) presented an expression for f_{app} applicable to lower Reynolds number turbulent flow and circular cross section ducts. It is valid for $2{,}300 < Re < 30{,}000$.

$$f_{app} = A\,Re^B \tag{11}$$

$$A = 0.09290 + \frac{1.01612}{(x/D_h)} \tag{12}$$

$$B = -0.26800 - \frac{0.31930}{(x/D_h)} \tag{13}$$

Phillips (1990) recommends that for rectangular cross section ducts with aspect ratio $\alpha \leq 1$, the Reynolds number should be calculated using the laminar equivalent diameter according to Eq. (7).

2.3 Evaluation of the heat transfer coefficient

Shah & London (1978) presented an expression valid for laminar flow regime to calculate the fully developed Nu_∞ based on rectangular cross section duct hydraulic diameter with aspect ratio $\alpha \leq 1$

$$Nu_\infty = 7.541\,(1 - 2.610\alpha + 4.970\alpha^2 - 5.119\alpha^3 + 2.702\alpha^4 - 0.548\alpha^5) \quad (14)$$

For laminar flow regime with velocity and temperature profiles simultaneously developing within a flat duct with isothermal walls and uniform velocity and temperature profiles at the duct entrance, Stephan (1959) presented the following expression for Nu_m as a function of the parameter $x^* = (x / D_h)/(Re\ Pr)$. This equation is valid for $0.1 < Pr < 1,000$:

$$Nu_m = 7.541 + \frac{0.024 x^{*-1.14}}{1 + 0.0354\, Pr^{0.17}\, x^{*-0.64}} \quad (15)$$

An attempt was made in this work to evaluate the mean Nusselt number in the entrance region of rectangular cross section ducts with aspect ratio $\alpha \leq 1$, combining Eq. (14) and (15) in this form

$$Nu_m = Nu_\infty + \frac{0.024 x^{*-1.14}}{1 + 0.0354\, Pr^{0.17}\, x^{*-0.64}} \quad (16)$$

Experimental data for the mean Nusselt number with $Pr = 0.72$ (air at normal atmospheric condition) were presented by Shah & London (1978). They were compared to those obtained from Eq. (16) for the same aspect ratios (α) and dimensionless variable (x^*). The experimental data are always larger than those predicted by Eq. (16). It is important to notice, however, that the experimental values were obtained considering four active walls of the rectangular duct while the fin channels have only three heated walls (see Fig. 1).

According to Shah & Bhatti (1987) the fully developed Nusselt number for turbulent flow regime in rectangular ducts can be obtained from the circular duct correlations replacing the circular duct diameter by the hydraulic diameter or the laminar equivalent diameter in the Reynolds and Nusselt numbers calculations. Therefore, the Gnielinski (1976) equation for constant wall flux or constant wall temperature condition can be used with the circular duct diameter replaced by the hydraulic diameter or the laminar equivalent diameter. It is valid for $2,300 \leq Re \leq 5.0 \times 10^6$ and $0.5 \leq Pr \leq 2,000$

$$Nu_\infty = \frac{(f/2)(Re_{eq} - 1,000)\, Pr}{1 + 12.7\,(f/2)^{0.5}(Pr^{0.67} - 1)} \quad (17)$$

In this equation the Reynolds number (Re_{eq}) is based on the laminar equivalent diameter, Eq. (7), and f is the turbulent friction factor from Eq. (11) with A = 0.09290 and B = - 0.26800, the fully developed flow condition.

An approach of the mean Nusselt number at the inlet region was calculated from an expression valid for circular ducts and Pr = 0.7 with $Nu_m = h_m D_{eq} / k_f$ according to Shah & Bhatti (1987)

$$\frac{Nu_m}{Nu_\infty} = 1 + \frac{2.4254}{(x/D_{eq})^{0.676}} \quad (18)$$

3. RESULTS

Data calculated through the presented correlations were compared with the predictions and the experimental data obtained by Knight et al. (1992b). They did not consider the resistances associated to the contraction and expansion of the airflow at the inlet and outlet of the fin channels. Thus, they predicted volumetric flow rates larger than the experimental data. In this work the total pressure drop was calculated including the three kinds of head loss according to Eq. (19). This equation was solved using the Newton-Raphson method with the total pressure drop (ΔP_T) always equal to 124.5 N/m^2 as made by the authors in their experimental procedure

$$\Delta P_T - \left(\Delta P_c + \Delta P_f + \Delta P_e\right) = 0 \quad (19)$$

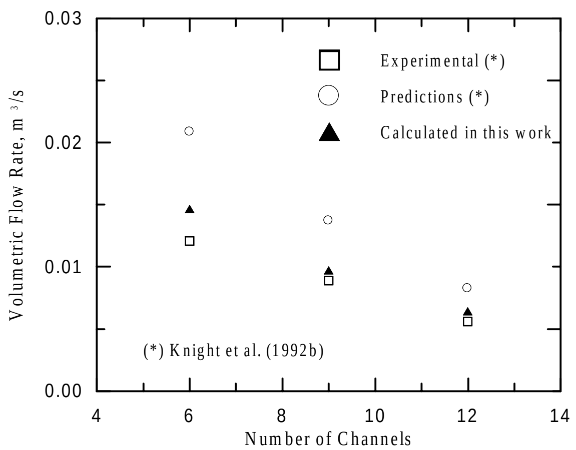

Fig. 7 – Volumetric flow rate versus the number of channels

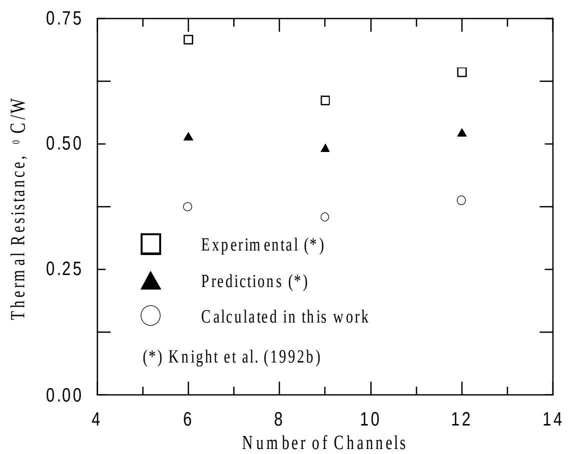

Fig. 8 - Thermal resistance versus the number of channels

The mean flow velocity in the fin channels and the thermal resistance of the fin array ($\theta = (T_w - T_i)/\dot{q}$) was determined considering a finite thermal conductivity of the fins material. The mean temperature at the base of fins is T_w, inlet fluid temperature is T_i and the heat rate dissipated by the fins is $\dot{q}$.

With the presented calculations for the heat sink with nine channels, the sudden contraction effect represented 31 % and the sudden expansion 7 % of the total pressure drop, so that these effects can not be ignored. The volumetric flow rates of the current predictions were about 9 % larger than the experimental data (see Fig. 7).

The thermal resistance values calculated in this work were closer to the experimental values due to the better volumetric flow rates predictions (see Fig. 8).

The three fin arrays tested were observed operating in turbulent flow regime. Nevertheless, the correlations for laminar flow regime were used by Reis (1998) considering the flow bypass phenomenon with good results.

4. CONCLUDING REMARKS

The thermal behavior of airflow cooled finned heat sinks was predicted using a network flow model and heat transfer correlations adapted from the literature. Results of the developed model were compared with experimental data and prediction values obtained in the literature. From this comparison, the model proposed in this work showed a better agreement with the experimental data than the predictions of the literature. There is, however, to extend the tests of this model to other comparisons.

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